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Author	Tuomas Pentinpoika Hytönen
Name of the dissertation *Translation-invariant operators on spaces of vector-valued functions*	
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Department Laboratory Field of research Opponent Instructor and supervisor	Engineering Physics and Mathematics Institute of Mathematics Fourier analysis, functional analysis Prof. Peter Sjögren (Göteborg) Prof. Stig-Olof Londen

Abstract: The treatise deals with translation-invariant operators on various function spaces (including Besov, Lebesgue–Bôchner, and Hardy), where the range space of the functions is a possibly infinite-dimensional Banach space X. The operators are treated both in the convolution form $Tf = k * f$ and in the multiplier form in the frequency representation, $\widehat{Tf} = m\hat{f}$, where the kernel k and the multiplier m are allowed to take values in $\mathcal{L}(X)$ (bounded linear operators on X). Several applications, most notably the theory of evolution equations, give rise to non-trivial instances of such operators.

Verifying the boundedness of operators of this kind has been a long-standing problem whose intimate connection with certain randomized inequalities (the notion of "R-boundedness" which generalizes classical square-function estimates) has been discovered only recently. The related techniques, which are exploited and developed further in the present work, have proved to be very useful in generalizing various theorems, so far only known in a Hilbert space setting, to the more general framework of UMD Banach spaces.

The main results here provide various sufficient conditions (with partial converse statements) for verifying the boundedness of operators T as described above. The treatment of these operators on the Hardy spaces of vector-valued functions is new as such, while on the Besov and Bôchner spaces the convolution point-of-view taken here complements the multiplier approach followed by various other authors. Although general enough to deal with the vector-valued situation, the methods also improve on some classical theorems even in the scalar-valued case: In particular, it is shown that the derivative condition

$$|\xi|^{|\alpha|} |D^\alpha m(\xi)| \leq C \qquad \forall \alpha \in \{|\alpha|_\infty \leq 1\} \cap \{|\alpha|_1 \leq \lfloor n/2 \rfloor + 1\}$$

is sufficient for m to be a Fourier multiplier on $L^p(\mathbb{R}^n)$, $p \in \,]1, \infty[$—the set of required derivatives constitutes the intersection of the ones in the classical theorems of S. G. Mihlin and L. Hörmander.

Keywords: Operator-valued Fourier multiplier; singular convolution integral; Lebesgue–Bôchner, Hardy, and Besov spaces of vector-valued functions; Mihlin's theorem; Hörmander's integral condition; R-boundedness; UMD space; Fourier embedding; evolution equation; maximal regularity.

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